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SIMPLIFIED MATHEMATICAL MODEL for GENERATING ECG SIGNAL and FITTING THE MODEL USING NONLINEAR LEAST SQUARE TECHNIQUE

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ABSTRACT
ECG models are complex and their computational time is high. In this paper, we propose a Gaussian wave-based model which can simulate ECG wave as well as P, Q, R, S and T waves individually. In addition, this model is capable of simulating various kinds of practical phenomena. The coefficient of the model was calculated by nonlinear least square technique using Gauss-Newton algorithm. In order to evaluate the effectiveness of the model, different kind time domain and frequency domain techniques such as PSD and MSC were used. The goodness of fitting was calculated using MSE, NMSE, RMSE, NRMSE and PRD and compared with real and model ECG signal. The lower value of these error and higher cross-correlation coefficient of 0.9208 between model and real ECG indicates the outstanding performance of the model. The model is also successful in generating noisy ECG signal.

Keywords: ECG Signal, Gaussian Wave, Nonlinear Least Square Technique, Goodness Of Fitting

1. INTRODUCTION

The heart is a hollow muscular organ that beats in rhythm to generate the force for pumping blood through the whole body. Each beats triggered by a bioelectric signals originated at sinoatrial (SA) node and spreads throughout the body. And the electrocardiogram (ECG or EKG) is an investigative tool for a wide range of heart conditions, from minor to life threatening that measure and records the electrical activity of the heart in superb detail. So ECG signal modeling and processing is the most significant topics in biomedical engineering.

For modeling of ECG, different techniques have been developed in the past. A pole-zero models of the ECG was represented by [1] for feature extraction and data compression. Another research [2] reported that, the poles and zeros form clusters and the clusters can be related to the constituent waves of the ECG models. Transform-type methods like nonlinear transform using multiplication backward difference for detecting QRS proposed by different researchers. However, this types of modeling cannot provide a direct representation of the constituent waves in the ECG as cardiac specialist are needed for making diagnoses.

Chip Away Decomposition (ChAD) algorithm which is an iterative method for Gaussian parameter determination was used for decomposing and representing the ECG model by [3]. Clifford et al. [4] used seven Gaussian functions for modeling of ECG by means of 3D state-space model which require numerical integration using a fourth-order Runge-Kutta method. S. Paravena et al. [5] used a large number of Gaussian (4 to 133) with no base line drift factor based on minimum bank method and zero crossing method. But fitting this model to the real ECG signal, starting and end point of any interval using zero crossing method is not efficient. In addition, growing number of Gaussian functions involve much time to run the program. [6] proposed a model using Gaussian function. However they cannot represent QRS wave individually as well as it is unable to fit with the real ECG at a significant level. They used double differentiation of the Gaussian function which is time consuming and need complex mathematical operation. The fitting techniques were incompetent because they were not capable to fit any negative values in their model which was quite common in real data.

This paper propose a Gaussian wave base model which can simulate ECG wave as well as P, Q, R, S and T wave individually and is very simple as compared to earlier mentioned model. Nevertheless, ECG signals are corrupted by various kinds of noises like other electrical signal, , such as (i) power lines interference [7], (ii) high-frequency electromyography (EMG) noise, (iii) motion artifacts, (iv) impedance changes at the skin/electrode, (v) baseline drifts [8], (vi) electrosurgical noise [9], and (vii) white noise. Therefore, noisy ECG is important as normal ECG to generate realistic ECG signal. In addition, noisy ECG can be generated by the model. The coefficient of the model is calculated section by section by nonlinear least square technique using gauss-Newton algorithm. The performance of the proposed model and fitting algorithm are evaluated by using ECG from recorded data. Thereby, for illustrating model performance, goodness of fitting are calculated. Real data are pre-processed for better fitting by using Butterworth filtering .Finally; the frequency-domain analysis of ECG signals is demonstrated.

The rest of this paper is organized as follows. Section 2 introduces the proposed model ECG model. Details methods of the proposed model are presented in Section 3. Section 4 provides experimental results and features of the model and conclusion and future work come in Section 5.

2. ECG MODEL

ECG signal is resembling of the combination of bell curve like P, Q, R, S and T waves; it falls toward both sites which are one of the characteristic of Gaussian wave. If $i \in (P, Q, R, S, T)$ then Gaussian wave for each component of ECG wave have following parameter: M_i is height of curves peak, t_i is the center position of the peak and W_i controls the width then ECG components can be written as

$$\text{P wave}: M_P\, e^{-\left(\frac{t-\tau_P}{\sqrt{2}W_P}\right)^2} \tag{1}$$

$$Q\ \text{wave}: M_{Q_1} e^{-\left(\frac{t-\tau_{Q1}}{\sqrt{2}W_{Q1}}\right)^2} + M_{Q_2} e^{-\left(\frac{t-\tau_{Q2}}{\sqrt{2}W_{Q2}}\right)^2} \tag{2}$$

$$R\ \text{wave}: M_R \frac{d}{dt} e^{-\left(\frac{t-\tau_R}{\sqrt{2}W_R}\right)^2} \tag{3}$$

$$S\ \text{wave}: -M_S\ e^{-\left(\frac{t-\tau_S}{\sqrt{2}W_S}\right)^2} \tag{4}$$

$$T\ wave: M_T\ e^{-\left(\frac{t-\tau_T}{\sqrt{2}W_T}\right)^2} \tag{5}$$

Simply the general equation (1-5) can be written as

$$f_i = \sum_{i \in P,R,S,T} \left(\pm \frac{d}{dt}\right)^i M_i e^{-\left(\frac{t-t_i}{\sqrt{2}w_i}\right)^2} + \sum_{i \in Q,\, j=1}^{j=2} M_{ij} e^{-\left(\frac{t-t_{ij}}{w_{ij}}\right)^2} \tag{6}$$

$$+ N_{j,SNR}(t)$$

In Eq. (6), $\left(\pm \frac{d}{dt}\right)^i$ depend on *i*.

$\left(\pm \frac{d}{dt}\right)^i = \left(\frac{d}{dt}\right)$ *iff* $i \in R$ and $\left(\pm \frac{d}{dt}\right)^i = (-1)$ *iff* $i \in S \cdot$

Otherwise the term does not exist.

If we perform normal sum of Eq. (6), the individual P, Q, R, S, T component of ECG will be overlapped with each other wave. To solve this problem, ECG components are fitted to their right position using shifting and zero padding method.

$N_{j,SNR}(t)$ is the noise parameter in the model. *j* indicates various types of ECG noise, such as white and color noise, muscle artifact (MA), electrode movement (EM), and baseline wander (BW). SNR indicates the input signal-to-noise ratio.

As real ECG is contaminated by these noises, so noise should be taken into account for more realistic modeling. Various noises are modeled in the following session.

(a)Synthetic noises

White noise is a random signal with a flat power spectral density. It has all frequency components. Flicker noise or color noise is a type of low-frequency electronic noise with an inversely proportional power spectral density compared with the frequency. Resistance fluctuation is the main reason for flicker noise generation and that's why all resistors has flicker noise. For the current study, we have modeled the noise color by a single parameter representing the slope of a spectral density function that decreases monotonically with frequency by following Eq. [10]

$$S(f) \propto \frac{\sigma^2}{f^{\beta}} \tag{7}$$

Where f is the frequency, σ^2 is the variance of the original signal and β is the slope; a measure of noise color. White noise (β=0), pink or flicker noise (β=1), and brown noise or the random walk process (β=2), are three of the most commonly referenced noises .White noise and color noise are simulated having 3dB input SNR using MATLAB.

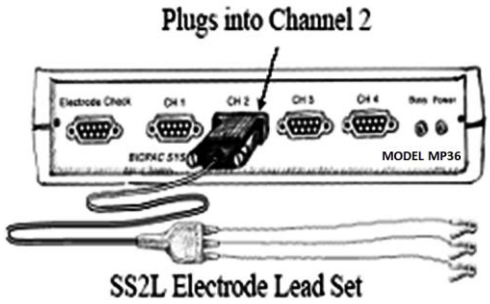
(b) Real Noise

Real noises are extracted from the noise stress test database in MIT-BIH [11]. Low-amplitude muscle noise is common in ECG. Muscle noise is, in contrast to baseline wander and 50/60 Hz interference. Mechanical movement of recording electrodes or skin stretching results in changes in potential. Due to alteration in the physical dimensions of the electrode-skin half cell thus modifying cell potential and skin-electrode impedance [12]. The proposed model is tested on the ECG data recorded from BIOPAC data acquisition system [13]. The noisy signal was generated by adding BW, EM, and MA artifacts (Noise Stress Test Database of MIT-BIH) and white and color noise were added to the clean ECG signals [12].

3. MATERIALS AND METHODS

3.1 ECG

The experiment is performed to collect the data for this research work. The subject is a male of 26 years old with no known cardiovascular disorder. For ECG and measurement, required equipments are BIOPAC electrode lead set (SS2L), BIOPAC disposable vinyl electrodes (EL503), BIOPAC data acquisition unit (MP36) [12] with cable and power. For ECG measurement, white lead was placed on right forearm, red lead on the left leg and the black lead was placed on right leg as shown in Fig. 2. Subjects was seated in a chair relaxing and asked to be as still as possible to ensure lower motion artifact and EMG signal on the data. After running calibration sequence ECG data was recorded.



(a)

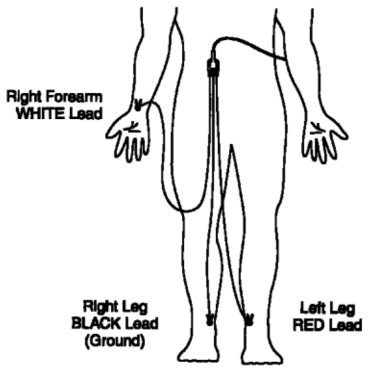


Fig 1. (a)MP36 Biopac (b)Placement of lead in ECG measureemnt.

3.2 Preprocessing

The acquired ECG data are preprocessed to remove noise, artifacts, and baseline wander using Savitzky-Golay Filtering [13]. To this end, two frequency-selective fourth-order Butterwort filters [14] are used: one high-pass filter with cutoff frequency at 0.5Hz and one low-pass filter with cutoff frequency at 90 Hz. To suppress the interferences from the power line grid, a notch filter centered around 50 was used. Again, this filter is implemented as a fourth-order Butterworth filter like [15].

As from the properties of Gaussian wave it is known that, the wave do not cross the zero. But P, Q, R, S and T waves cross the zero line in the real ECG signal. To solve this zero crossing problem 0.05 was added to real ECG signal before calculating the coefficients, so that the baseline of real ECG lies in zero line.

3.3 Nonlinear Fitting

For fitting the mathematical model with the real world data statistical hypothesis testing like: test of normality of residuals, chi square test, analysis of variance, least square test etc [16] is needed. As it observed that, ECG model is nonlinear in the coefficients. So the nonlinear least square techniques can be the best choice for this fitting. Nonlinear models are more difficult to fit than linear models because the coefficient cannot be estimated using simple techniques instead an iterative approaches required to solve this problem.

Consider that an ECG function $y=f(x)$ of a variable of x tabulated at i values, where $y_1=f(x_1)$, $y_2=f(x_2)\ldots\ldots\ldots\ldots y_i=(x_i)$. Moreover, assume that the known analysis form the function depending on j parameters $f(x;\phi_1,\phi_2,\ldots\ldots\ldots,\phi_j)$ and the set of i equation will be

$$\left.\begin{array}{l} y_1 = f(x_1;\phi_1,\phi_2,\ldots\ldots\ldots\ldots\phi_j) \\ y_2 = f(x_2;\phi_1,\phi_2,\ldots\ldots\ldots\ldots\phi_j) \\ \qquad\vdots \\ y_i = f(x_i;\phi_1,\phi_2,\ldots\ldots\ldots\ldots\phi_j) \end{array}\right\} \tag{8}$$

We have to solve the equation to obtain the value of $\phi_1,\phi_2,\ldots\ldots\ldots,\phi_j$ which satisfies our model properly. At first an initial value is picked for ϕ_k and defined

$$d\beta_k = y_k - f(x_{k;}\phi_1,\phi_2,\ldots\ldots\ldots,\phi_j) \tag{9}$$

And then the estimation for the change $d\phi_k$ needed to reduce $d\beta_k$ to 0

$$d\beta_k = \sum_{l=1}^{j} \frac{df}{d\phi_l} d\phi_l \bigg|_{x_k\phi} \tag{10}$$

For $k=1,2,\ldots.,i$ where $\phi \equiv (\phi_1,\phi_2,\ldots,\phi_j)$

This element can be written as a $i \times j$ matrix of partial derivatives of

$$A_{kl} = \begin{vmatrix} \left.\frac{df}{d\phi_1}\right|_{x_1\phi} & \ldots. & \ldots. & \ldots.. & \left.\frac{df}{d\phi_j}\right|_{x_1\phi} \\ \left.\frac{df}{d\phi_1}\right|_{x_2\phi} & \ldots. & \ldots. & \ldots. & \left.\frac{df}{d\phi_j}\right|_{x_2\phi} \\ \vdots & \ddots & & & \\ \vdots & & \ddots & & \\ \left.\frac{df}{d\phi_1}\right|_{x_i\phi} & \ldots. & \ldots. & \ldots. & \left.\frac{df}{d\phi_j}\right|_{x_i\phi} \end{vmatrix} \tag{11}$$

Then,

$$d\beta_k = A_{kl} d\phi_l \tag{12}$$

And the brief equation is

$$d\beta = A d\phi \tag{13}$$

If by defining $a=A^TA$ and $b=A^T d\beta$

We find,

$$a\, d\phi = b \tag{14}$$

Then Eq. (14) is solved for $d\phi$ using Gaussian elimination techniques. This offset is applied to α and a new $d\phi$ is calculated. By interactively applying this procedure until the elements of $d\phi$ become smaller than desired limit, a solution is obtained. The sum of square residuals is calculated by $R^2 = d\beta.d\beta$ after the final iteration.

3.5 Performance Evaluation Parameters

3.5.1 Coherence

The magnitude squared coherence (MSC) estimate is a function of frequency with values between 0 and 1 that indicates how well the model ECG corresponds to real ECG at each frequency. The MSC estimate C_{xy} of the input signals (x and y) using Welch's averaged, modified periodogram method [17]. The MSC is nothing but a function of the power spectral densities ($P_{xx}(f)$ and $P_{yy}(f)$) of x and y and the cross power spectral density ($P_{xy}(f)$) of x and y.

$$C_{xy} = \frac{|P_{xy}(f)|^2}{P_{xx}(f)P_{yy}(f)} \tag{15}$$

In this paper; x, y represents the model and real ECG signal respectively and x and y must be the same length.

3.5.2 Power spectral density (PSD)

PSD represents the strength of the energy as a function of frequency [17].In other words, it shows at which frequencies variations are strong and at which frequencies variations are weak. Energy can be obtained within a specific frequency range by integrating PSD within that frequency range. Computation of PSD is done directly by the method called FFT or computing autocorrelation function and then transforming it.

3.5.3 Cross-correlation coefficient

If x(n) be the recorded or collected ECG signal and $x_m(n)$ be the ECG signal generated by the mathematical model, then cross-correlation coefficient ρ between $x(n)$ and $x_m(n)$ is given by [18]

$$\rho = \frac{\langle [x(n)-\mu_x][x_m(n)-\mu_m] \rangle}{\sigma_x \sigma_m} \tag{16}$$

Where $\langle \bullet \rangle$ denotes the average calculated by summing over the observed time series, indexed by n. where μ_x and σ_x are the mean and standard deviation of *x(n)*, and μ_m and σ_m are the mean and standard deviation of $x_m(n)$. A value of ρ ~ 1 reflects a strong correlation, ρ ~−1 implies a strong anticorrelation, and ρ ~ 0 indicates that *x(n)* and $x_m(n)$ are uncorrelated. This means that a value of ρ = 1 suggests that model and real ECG are identical.

3.5.4 Error Assessment:

The Mean Square Error (MSE) is defined as [19].

$$MSE = \frac{1}{N}\sum_{n=0}^{N-1}[x(n) - x_m(n)]^2 \tag{17}$$

The normalized form of MSE is

$$NMSE = \frac{\sum_{n=0}^{N-1}[x(n) - x_m(n)]^2}{\sum_{n=0}^{N-1}[x(n)]^2} \tag{18}$$

Another measurement is Root Mean Square Error, which is

$$RMSE = \sqrt{\frac{1}{N}\sum_{n=0}^{N-1}[x(n) - x_m(n)]^2} \tag{19}$$

The Normalized version of RMSE is

$$NRMSE = \sqrt{\frac{\sum_{n=0}^{N-1}[x(n) - x_m(n)]^2}{\sum_{n=0}^{N-1}[x(n)]^2}} \tag{20}$$

Percent Root Mean square Difference (PRD) can be determined by Eq. (26).

$$PRD = \sqrt{\frac{\sum_{n=0}^{N-1}[x(n) - x_m(n)]^2}{\sum_{n=0}^{N-1}[x(n)]^2}} \times 100\% \tag{21}$$

4. RESULT AND DISCUSSION

The coefficient of the model is calculated by nonlinear least square technique using Gauss-Newton algorithm having 95% confidence level and the coefficient are shown in Table 1.

Table 1: Coefficient to fit the model with BIOPAC recorded ECG

ECG Component		Coefficient		
		A_i	B_i	t_i
P		0.185	17.8	236.9
Q	j=1	-0.1103	30.64	26.8
	j=2	-0.1075	5.705	76.23
R		50	29.87	55.71
S		0.509	9.09	11.9
T		0.3255	29.78	154.3

Using the value of the model coefficient, the real and model ECG are quite similar. This is represented in fig 2.

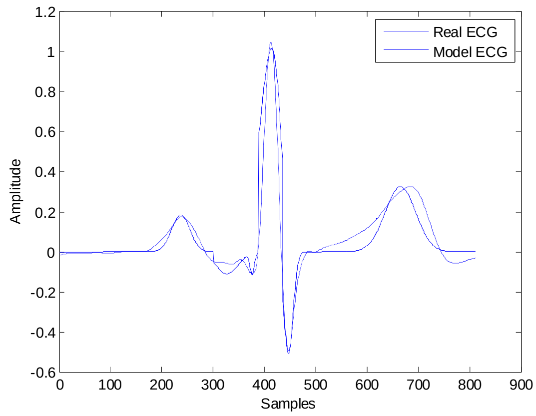


Fig 2. Comparison between real and model ECG signal

To evaluate the performance of the model, goodness of fitting was calculated using Eq. (17) ~ Eq. (21) and the values are given in Table 2. It is observed that, the values of different errors were quite low and the value of cross-correlation coefficient was 0.9205 which indicated the highly correlation between the real and model ECG.

Table 2: Goodness of fitting

Goodness of fitting	**Value** (BIOPAC)
MSE	0.00779
NMSE	0.172477
RMSE	0.0882615
NRMSE	0.029748
PRD	2.974839

The model was not only evaluated in the time domain but also in the frequency domain. Figure 3 and Fig. 4 showed the power spectral density (PSD) and magnitude square coherence (MSC) respectively. In Fig 3, the model and real ECG are reasonably similar and the MSC in fig 4, was exists 1 in most of the frequencies which is a better indication of the proposed model.

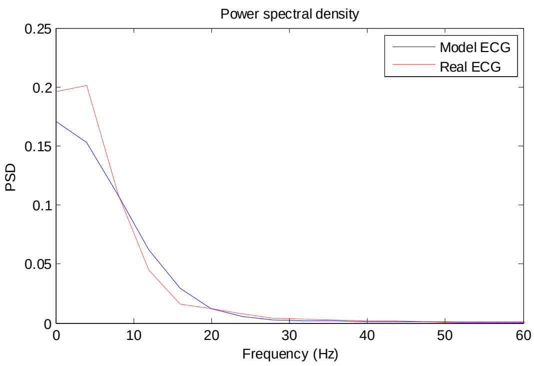


Fig 3. Comparison of power spectral density between real and model ECG signal.

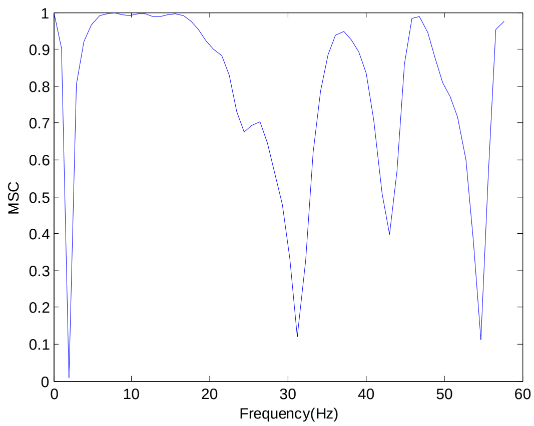


Fig 4. Magnitude squared coherence (MSC) between real and model ECG signal.

The model not only simulate ECG beat but can simulate different heart condition like Brachycardia (Slow heart rate), Tachycardia (Fast heart rate) etc. From Fig. 5, brachycardia was simulated for 5 sec possessing 4 beat. So the beat per minute (BPM) was 48. Sinus rhythm and tachycardia were simulated in the same way.

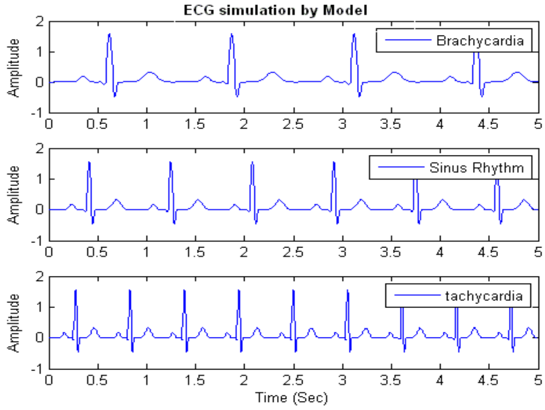


Fig 5. ECG simulation by model for 5 Sec. For brachycardia with BPM 48, sinus rhythm with BPM 72, tachycardia with BPM 108.

Another feature of the model is that, it can generate realistic and simulated noisy ECG signal. $N_{j,SNR}(t)$ of Eq. (6) indicates the noise parameter of the model. Different noisy ECG signal (different *j*) were simulated for the input SNR of 3dB in fig 6.

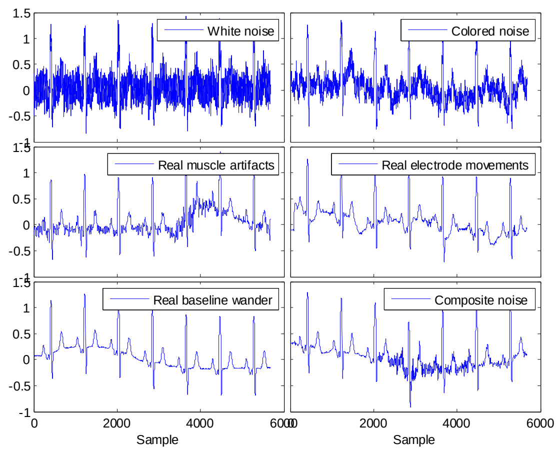


Fig 6. Different types of noisy ECG signal.

5. CONCLUSION

The proposed model is capable of replicating many important features of the human ECG wave. A number of features and applications of the model are:

- Instead of using too many parameters like other, the proposed system can generate ECG and capable of simulating various kind of practical phenomena such as brachycardia, tachycardia.
- It does not need three dimensional state spaces which are difficult for realization and simulation.
- Noisy ECG signal can be modeled by simply adding a noise parameter with the model.
- In frequency domain the real and modeled ECG showed almost same properties.
- A realistic ECG database can be created by

fitting the model with individual subject's ECG. This can be used for further analysis and for education purpose.

- By only saving the coefficient of the model, ECG data compression can be possible.

However, the lower value of error parameters like MSE 0.00779 indicates the effectiveness of this simplified model and the proposed model enables us to investigate wave morphology variation. It can be improved for better fitting, model-based denoising, compression and neural network based classification etc which is under investigation. Finally, it is hoped that this simple model will provide an efficient tool for testing and processing of the ECG signals with different level of noise and/or motion artifact.

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